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MODELS OF PRIMARY INFORMATION SENSORS OF EMERGENCY EFFECTS MITIGATION SYSTEMS

Mathematical models of the primary information sensors based on the Hankel and Laplace integral transforms is built. The transforms are applied for solving Fourier differential equations.

Keywords: Fourier equation, heat flow, primary information sensor.

Problem formulation. Emergency effects mitigation systems provide significant reduction of the damage caused by emergencies [1]. There are many factors influenced on the efficiency of the emergency effects mitigation system. One of them is adequacy of the mathematical description of the processes occurring in the primary information sensors of the systems. In this regard, one of the problems of creating the emergency effects mitigation systems is creating and improving the database of mathematical support of the systems and their components for the various stages of their life.

Analysis of recent researches and publications. There are many mathematical models of the primary information sensors [2]. Their form and parameters are defined by the objective function and physical processes in the sensors. Mathematical description of physical processes in the primary information sensors for emergencies associated with the release of heat energy are presented in generalized form in [3, 4]. It should be emphasized that all these mathematical models of sensors describe the process of their normal functioning and as a rule they don’t focus on the features of their work during various kind of tests. It is known mathematical models of the thermo-resistive sensors focused on the sensor features in temperature tests [5]. However these models can be applied only in the cases with Joule-Lenz effect. For example, the models couldn’t be used for the sensor test method based on the formation the stationary heat flow supplied to the thermo-resistive element of the sensor.

Statement of the problem and its solution. The main goal of the work is to build mathematical models describing processes in the primary information sensor of the emergency effects mitigation system under stationary heat flow.

We assume that sensor of the primary information detector is made as solid cylinder of radius R and its height is much more then radius. This sensor is impacted by stationary heat flow q. Temperature distribution in the sensor is described by Fourier equation

$$\frac{\partial \theta(r,t)}{\partial t} = a\left[\frac{\partial^2 \theta(r,t)}{\partial r^2} + \frac{1}{r}\cdot\frac{\partial \theta(r,t)}{\partial r}\right] - m^2\theta(r,t), \tag{1}$$

with initial and boundary conditions

$$\theta(r,0) = 0;\ \frac{\partial\theta(R,t)}{\partial r} = \frac{q}{\lambda}, \tag{2}$$

$\theta(r,t) = T(r,t) - T_0$ is temperature of sensor; T_0 is ambient temperature; a is thermal diffusivity coefficient; λ is thermal conductivity of the sensor material; m^2 is the parameter defined by expression

$$m^2 = \frac{2\alpha}{c\rho R}, \tag{3}$$

α is heat transfer coefficient; c, ρ are specific heat capacity and density of the sensor material.

Let introduce the notation

$$\theta(r,t) = M(r,t)\exp(-m^2 t), \tag{4}$$

therefore (1) and (2) we transform as follows

$$\frac{\partial M(r,t)}{\partial t} = a\left[\frac{\partial^2 M(r,t)}{\partial r^2} + \frac{1}{r}\cdot\frac{\partial M(r,t)}{\partial r}\right], \tag{5}$$

$$M(r,0) = 0;\ \frac{\partial M(R,t)}{\partial r} = \frac{q}{\lambda}\exp(m^2 t). \tag{6}$$

Applying the Hankel integral transform [6] to the expression (5) gives

$$\bar{f}(\mu_n, t) = \int_0^R r J_0\left(\frac{\mu_n r}{R}\right) f(r,t)\,dr. \tag{7}$$

To regard the condition (6) it gives

$$\frac{d\bar{M}(\mu_n,t)}{dt} + a\left(\frac{\mu_n}{R}\right)^2 \bar{M}(\mu_n,t) = \frac{aq}{\lambda} J_0(\mu_n)\exp(m^2 t), \tag{8}$$

μ_n – the n-th root of the transcendental equation

$$J_1(\mu) = 0, \tag{9}$$

J_0, J_1 are Bessel functions of zero and first order.

Let's apply Laplace integral transform to the differential equation (8). It gives [7]

$$\overline{\overline{M}}(\mu_n, p) = \frac{aqR}{\lambda} J_0(\mu_n) \left[\left[p + a\left(\frac{\mu_n}{R}\right)^2 \right] \left(p - m^2\right) \right]^{-2}, \tag{10}$$

where

$$\overline{\overline{M}}(\mu_n, p) = \int_0^\infty \overline{M}(\mu_n, t) \exp(-pt) dt. \tag{11}$$

Then solution of the differential equation (8) has the form [8]

$$\begin{aligned} \overline{M}(\mu_n, t) = L^{-1}\left[\overline{\overline{M}}(\mu_n, p)\right] = \frac{aqR}{\lambda} J_0(\mu_n) \left[a\left(\frac{\mu_n}{R}\right)^2 + m^2 \right]^{-1} \times \\ \times \exp(-m^2 t) \left[1 - \exp\left[-\left[a\left(\frac{\mu_n}{R}\right)^2 + m^2 \right] t \right] \right], \end{aligned} \tag{12}$$

L^{-1} is the operator of the inverse Laplace integral transform.

According to inversion formula [6] the solution of the differential equation (1) determining the temperature distribution in a fire detector sensor is

$$f(r, t) = \frac{\bar{f}(\mu_0)}{\|\psi_0\|^2} + \sum_{n=1}^{\infty} \frac{J_0\left(\frac{\mu_n r}{R}\right) \bar{f}(\mu_n, t)}{\|\psi_n\|^2}, \tag{13}$$

where

$$\|\psi_n\|^2 = \begin{cases} \frac{R^2}{2}, n = 0; \\ \frac{R^2}{2} J_0^2(\mu_n), n \geq 1, \end{cases} \tag{14}$$

$\mu_0 = 0$. In according to (4) it means

$$\theta(r,t) = \frac{2aq}{\lambda m^2 R}\left[1 - \exp(-m^2 t)\right] + \\ + \frac{2aq}{\lambda R}\sum_{n=1}^{\infty}\frac{J_0\left(\frac{\mu_n r}{R}\right)}{J_0(\mu_n)}\left[a\left(\frac{\mu_n}{R}\right)^2 + m^2\right]^{-1}\left[1 - \exp\left[-\left[a\left(\frac{\mu_n}{R}\right)^2 + m^2\right]t\right]\right]. \quad (15)$$

Averaging (15) by sensor volume and using (3), (9) give

$$\theta(t) = \frac{2}{R^2}\int_0^R r\theta(r,t)dr = \frac{q}{\alpha}\left[1 - \exp\left(-m^2 t\right)\right] \quad (16)$$

Expression (16) describes reaction of fire detector sensor to the impact of stationary heat flow which value is equal to q. The dynamic properties of this process are completely characterized by a time constant. In according to (16) and (3) it is

$$\tau = \frac{0,5c\rho R}{\alpha}. \quad (17)$$

Parameter α is determined from criteria equation [9]. It gives

$$\alpha = d_1\lambda_0\ell^{-1,0}\,\mathrm{Re}^{d_2}\,\mathrm{Pr}_1^{d_3}\left(\frac{\mathrm{Pr}_1}{\mathrm{Pr}_2}\right)^{0,25}, \quad (18)$$

λ_0 is the thermal conductivity of air; ℓ is the length of the sensor of a primary information detector; Re is Reynolds number; Pr_1 is Prandtl number for the temperature of airflow; Pr_2 is Prandtl number for the temperature of sensor surface; d_i are parameters defined by value of Reynolds number Re (see the table 1).

Tab. 1. Values of the parameters d_i

Reynolds number	d_1	d_2	d_3
$5 < \mathrm{Re} < 10^3$	0,5	0,5	0,38
$10^3 < \mathrm{Re} < 2\cdot10^5$	0,25	0,6	0,38
$2\cdot10^5 < \mathrm{Re} < 2\cdot10^6$	0,023	0,8	0,37

To a first approximation it can be assumed $\mathrm{Pr}_1 = \mathrm{Pr}_2$.

Conclusions. It is shown that impact of stationary heat flow to the sensor of a primary information detector of the emergency effects mitigation system characterizes by time constant in dynamic mode. Value of the time constant is determined by rheological, geometric and thermal characteristics. It can be used to form the thermal test algorithm for the same kind detectors.

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Моделі датчика первинної інформації системи ослаблення наслідків надзвичайних ситуацій

Отримано математичні моделі датчика первинної інформації, в основі яких лежить використання інтегральних перетворень Ханкеля і Лапласа стосовно до вирішення диференціального рівняння Фур'є.

Ключові слова: рівняння Фур'є, тепловий потік, датчик первинної інформації.

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Модели датчика первичной информации системы ослабления последствий чрезвычайных ситуаций

Получены математические модели датчика первичной информации, в основе которых лежит использование интегральных преобразований Ханкеля и Лапласа применительно к решению дифференциального уравнения Фурье.

Ключевые слова: уравнение Фурье, тепловой поток, датчик первичной информации.